Practical Assignment

Answer in no more than 10 pages total
Minimum 10pt font size

October 27, 2014

1. (**Loopback test**) Obtain an audio cable with 3.5mm stereo jacks on either end. Connect one end to the audio output of your soundcard, and the other end to the audio input. Using the software available on the course website (see folder called "loopback"), or your own software, play a sinusoidal signal of frequency 100 Hz for a finite duration of time (atleast 2 seconds). Assert that you can hear the tone when the cable is not plugged in and the audio is played through internal computer speakers, or a set of connected headphones. Obtain samples at rate $F_s = 44\,100\,\text{Hz}$,

$$x_1, \dots, x_L, \qquad y_1, \dots, y_L$$

   from the left and right channels of the soundcard input, where L is the number of samples obtained. Reconstruct the signals as

$$\tilde{x}(t) = \sum_{\ell=1}^{L} x_\ell \operatorname{sinc}(F_s t - \ell), \qquad \tilde{y}(t) = \sum_{\ell=1}^{L} y_\ell \operatorname{sinc}(F_s t - \ell)$$

   where

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}. \tag{1}$$

   Plot these reconstructed signals for a 20 ms window from $t = 1\,\text{s}$ to 1.02 s.

   Using scissors or wire cutters cut one of the channels of your audio cable. Alternatively, cut both and reconnect one. Run the test again and plot the reconstructed signals for a 20 ms window from $t = 1\,\text{s}$ to 1.02 s.

2. (**Multiplier**) Consider the operational amplifier circuit in Figure 1. Analyse this circuit to obtain a relationship between the input voltage x and the output voltage y. Build the circuit on a breadboard and, using the soundcard, input the signal

$$x(t) = \tfrac{1}{3}\sin(2\pi f_1 t) + \tfrac{1}{3}\sin(2\pi f_2 t)$$

   with $f_1 = 100$ and $f_2 = 233$. Using the (stereo) soundcard simultaneously record the input signal x directly from the soundcard ouput and also the output voltage signal y. Build reconstructed approximate signals $\tilde{x}$ and $\tilde{y}$ from the samples obtained and hypothesise a relationship between $\tilde{x}$ and $\tilde{y}$. Plot $\tilde{x}$, $\tilde{y}$ and the hypothesised signal over a 20 ms duration and comment on the validity of your hypothesis. List the components that you used in constructing the circuit.

3. (**Band-pass filter**) Consider the operational amplifier circuit in Figure 2. Assuming the operational amplifier is ideal, find a differential equation relating the input voltage signal x with the output voltage signal y. Find the transfer function of the system H mapping x to y. Find the poles and zeros of the system and construct a pole-zero plot. Assert that H is stable and regular and find its impulse response h.

   Build the circuit on a breadboard and, using a computer soundcard, input the signal

$$x(t) = \tfrac{1}{3}\sin(2\pi f_1 t) + \tfrac{1}{3}\sin(2\pi f_2 t)$$

   with $f_1 = 500$ and $f_2 = 1333$. Using the (stereo) soundcard simultaneously record the input signal x and also the output voltage signal y. Build reconstructed approximate signals $\tilde{x}$ and $\tilde{y}$ from the samples obtained. Plot $\tilde{x}$, $\tilde{y}$ and $H(\tilde{x}) = h * \tilde{x}$ over a 4 ms duration. Assert that $h * \tilde{x}$ is close to $\tilde{y}$. To compute $h * \tilde{x}$ you may wish to use the trapezoidal integration method used in Test 5 of the lecture notes.

Now measure the spectrum of the circuit by using the soundcard to input sinusoidal signals of the form

$$x_k(t) = \sin(2\pi f_k t), \qquad f_k = 110 \times 2^{k/2}, \;\; k = 0, 1, \dots, 12.$$

For each $k = 0, 1, \dots, 12$ obtain an estimate of the spectrum $\Lambda(H, f_k)$. You may wish to use the method described in Test 4 of the lecture notes. Find an analytical expression for the spectrum of the system $\Lambda(H, f)$ and plot the magnitude and phase spectrum over the interval $f \in [0, 7500]$. On the same plots draw the measurements of the magnitude and phase spectrum obtained using the sound card. Assert that the measurements conform with the hypothesised spectrum $\Lambda(H, f)$. List the components used in constructing the circuit.

4. **(Butterworth filter)** Design a lowpass second order Butterworth filter with cuttoff frequency in the range 1800 Hz to 2200 Hz. Draw a diagram of the electrical circuit you have designed and list the components. Derive the transfer function and the spectrum of your filter. Construct the circuit and, using the computer soundcard, measure its spectrum over frequencies in the range 100 Hz to 7000 Hz. Plot your measurements alongside the hypothesised spectrum that you derived.

5. (**Filtering a SNES classic**) The file `classic.wav` available at:

   `http://robbymckilliam.github.io/signalsandsystems/2014/classic.wav`

   contains 13 seconds of audio from a classic super Nintendo game. The audio is in mono format and sampled at $F = 44\,100$ Hz. Added to the audio is an interfering sinusoid. Read this file using a computer and find the number of samples that this audio file contains. Denote the number of samples by N. Suppose that the original audio signal is bandlimited with bandwidth less than 22 050 Hz. Plot the magnitude of the Fourier transform of the signal at the frequencies

$$f_k = \frac{200k}{PL} \qquad k = 0, \dots, 2000$$

   where $L = 2^{\lceil \log_2 N \rceil}$ is the smallest power of two greater than or equal to N. In a similar manner to Test 10 from the lecture notes, use the fast Fourier transform to compute a signal with the interfering sinusoid removed. Plot the magnitude of the Fourier transform of this signal at the frequencies $f_k, k = 0, \dots, 2000$. Write samples to a wav file and confirm that the interfering sinusoid is no longer audible.

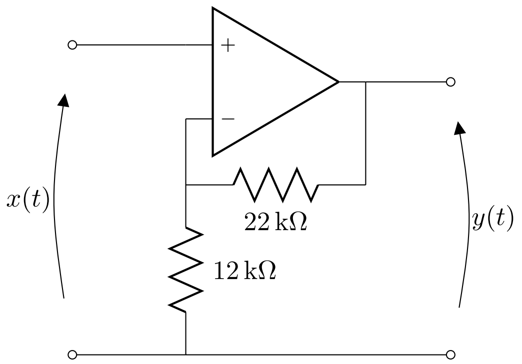

Figure 1: Operational amplifier circuit configured as a multiplier

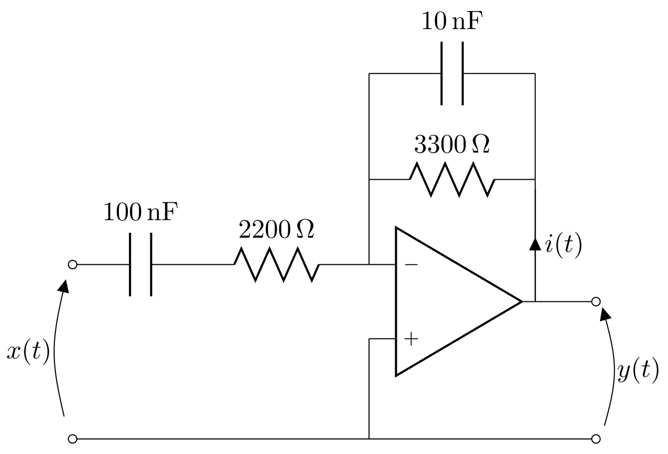

Figure 2: Operational amplifier configured as a band-pass filter with two capacitors and two resistors.