# Practical Assignment 2 

Answer in no more than 8 pages total Minimum 10pt font size

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1. (Band-pass filter) Consider the operational amplifier circuit in Figure 1 Assuming the operational amplifier is ideal, find a differential equation relating the input voltage signal $x$ with the output voltage signal $y$. Find the transfer function of the system $H$ mapping $x$ to $y$. Find the poles and zeros of the system and construct a pole-zero plot. Assert that $H$ is stable and regular and find and plot its impulse response $h$.
Build the circuit on a breadboard and measure the spectrum of the circuit by using the soundcard to input sinusoidal signals of the form

$$
x_{k}(t)=\sin \left(2 \pi f_{k} t\right), \quad f_{k}=110 \times 2^{k / 2}, \quad k=0,1, \ldots, 12
$$

For each $k=0,1, \ldots, 12$ obtain an estimate of the spectrum $\Lambda H\left(f_{k}\right)$. You may wish to use the method described in Test 4 of the lecture notes. Find an analytical expression for the spectrum of the system $\Lambda H(f)$ and plot the magnitude and phase spectrum over the interval $f \in[0,7500]$. At what positive frequency $f$ is the magnitude spectrum maximised? On the same plots draw the measurements of the magnitude and phase spectrum obtained using the sound card. Assert that the measurements conform with the hypothesised spectrum $\Lambda H(f)$. List the components used in constructing the circuit.
Now input the signal

$$
x(t)=\frac{1}{3} \sin \left(2 \pi f_{1} t\right)+\frac{1}{3} \sin \left(2 \pi f_{2} t\right)
$$

with $f_{1}=500$ and $f_{2}=1333$. Using the soundcard simultaneously record the input signal $x$ and also the output voltage signal $y$. Build reconstructed approximate signals $\tilde{x}$ and $\tilde{y}$ from the samples obtained. Plot $\tilde{x}, \tilde{y}$ and $H \tilde{x}=h * \tilde{x}$ over a 4 ms duration. Assert that $h * \tilde{x}$ is close to $\tilde{y}$. To compute $h * \tilde{x}$ you may wish to use the trapezoidal integration method used in Test 5 of the lecture notes.
2. (Butterworth filter) Design a lowpass second order Butterworth filter with cuttoff frequency in the range 2200 Hz to 2600 Hz . Draw a diagram of the electrical circuit you have designed and list the components. Derive the transfer function and the spectrum of your filter. Construct the circuit and, using the computer soundcard, measure its spectrum over frequencies in the range 100 Hz to 7000 Hz . Plot your measurements alongside the hypothesised spectrum that you derived.


Figure 1: Operational amplifier configured as a band-pass filter with two capacitors and two resistors.

