Theory Assignment

Answer in no more than 10 pages total
Minimum 10pt font size
Due Friday, 9 October 2015, 5:00 PM
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September 14, 2015

1. (**Multiplier**) Consider the operational amplifier circuit in Figure 1. Draw an equivalent circuit using the model for an operational amplifier including input resistance R_i, output resistance R_o and open loop gain A (given in Figure 2.5 of the lecture notes). Analyse this circuit to obtain a relationship between the input voltage signal x and output voltage signal y. By taking limits as $R_i \to \infty$, $A \to \infty$ and $R_o \to 0$ find an expression relating x and y assuming that the operational amplifier is ideal. Obtain the same expression directly using the rules for analysing ideal operational amplifiers. Is the system that describes this circuit stable? Is it regular?

2. (**DC motor and winch**) Suppose that a DC motor and winch is used to tow a body of mass M as depicted in Figure 2. The mass (or moment of interia) of the winch is negligibile compared with M. The relationship between the force f at the circumference of the winch and the turning force (torque) τ is $\tau = rf$ where r is the radius of the winch. The winch is connected to the mass by an elastic cable modelled as a spring with constant K. As the motor turns the length of the cable ℓ changes. The angle of the motor θ is related the length of the cable by $\ell = 2\pi r\theta$. The force due to the spring is modelled as $K(\ell - p)$. A force due to friction f_r opposses the motion of the mass so that $f_r = -B_r Dp$ where B_r is the coefficient of friction. Supposing that the inductance within the motor $L = 0$, find a differential equation that relates the input voltage v with the position of the mass p. Find the transfer function of this system. Is this system stable?

   Now assume that the constants satisfy

   $$r = B_r = R = K_t = K_b = M = K = 1.$$

   Under these assumptions: find the poles and zeros, draw a pole zero plot, and find and plot the impulse response.

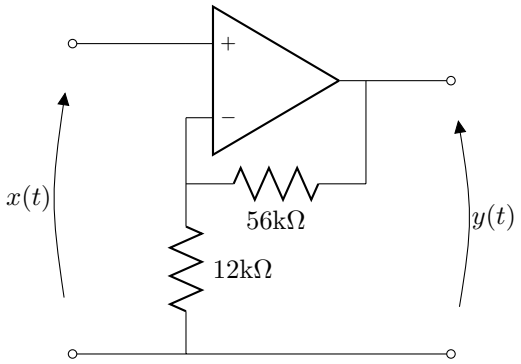

Figure 1: Operational amplifier circuit configured as a multiplier

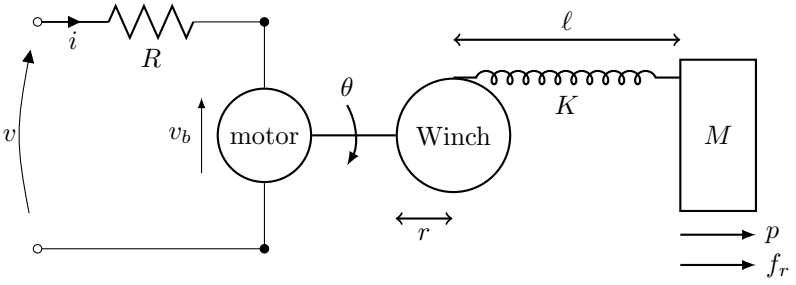

Figure 2: Winch, DC motor, and mass.